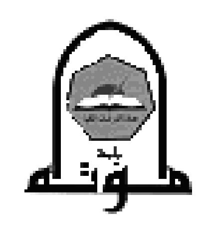


Mu'tah University
Deanship of the Graduate Studies

ON BERNSTEIN POLYNOMIALS AND APPROXIMATION

By

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DEDICATION

To my parents and my sisters with love and respect.

Shahrazad AL-Dalaeen

ACKNOWLEDGEMEN

"Praise be to Allah, Who has guided us to this (felicity): never could we have found guidance, had it not been for the guidance of Allah".

I would like to express my sincere appreciation to my supervisor Dr. Rateb AL-Btoush for his valuable assistance and continuous guidance.

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Shahrazad. N. Al-Dalaeen

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LIST OF SYMBOLS

Symbol	Meaning
E_n	The error of best approximation to $f(x)$ by the polynomials belonging to π_n.
$\omega(\delta)$	Modulus of continuity of f on I.
$\|\|.\|\|$	Norm.
$\|\|.\|\|_2$	L_2 norm.
$\|\|.\|\|_\infty$	The maximum norm or infinte norm.
Δ	Forward difference operator .
$p^* \in \pi_n$	Best approximation to $f(x)$ by the polynomials belonging to π_n.
L_n	Linear monotone operator.
$d(x, y)$	Distance from x to y.
$f \epsilon Lip$	f satisfy the Lipschitz condition.
$B_n(f; x)$	Bernstein polynomials.
$C[0,1]$	Space of Continuous functions on closed interval $[0,1]$.
ζ	Point in the open interval.
$p(t)$	Bézier curve.
$\mathbb{R}^d$	Euclidean space, such that $d = \{2.3\}$.
$\mathbb{R}$	Real line.
$\Delta_h^k f(x)$	$k\ th$-order forward difference of $f(x)$with step length h.
$\|f(x)\| \leq M$	Function f is bound .
$B_n(1)(x)$	The Bernstein polynomial of function $f = 1$.

$B_n(x)(x)$	The Bernstein polynomial of function $f = x$.
$B_n(x^2)(x)$	The Bernstein polynomial of function $f = x^2$.
$N \in \mathbb{N}$	Natural number.
$P_n(x)$	Sequence of polynomials.
D_i	Control points of the first derivative Bézier curve.
$j_{nB}(x)$	Jackson polynomial by algebraic polynomials (Bernstein polynomials).
$j_n(x)$	Jackson polynomial by trigonometric polynomials.
$R_n(x)$	Remainder term on Taylor series.
w_i	Weights of control points.
$R(t)$	Rational Bézier curves.
$C^0\ [a,b]$	Continuous function on the closed interval $[a,b]$.
$B(S)$	Class of functions which are bounded on S.
2D	Space in two dimention
CAD software	computer-aided design and drafting (CADD), is the use of computer technology for design and design documentation.

الملخص

حول كثيرات حدود بيرنشتاين والتقريب

شهرزاد نصر الضلاعين

جامعة مؤته 2015

في هذه الاطروحة تمت دراسة بعض خصائص كثيرات حدود بيرنشتاين واثباتها. ولقد تم استخدام بعض هذه الخصائص في برهان نظرية ويرستراس. وعلاوة على ذلك، تم دراسة نتائج اضافية متعلقة بكثيرات حدود بيرنشتاين والتي تستخدم في برهان نظرية جاكسون. ثم دراسة نتائج مماثلة باستبدال كثيرات الحدود المثلثية في نظرية جاكسون بكثيرات حدود (بيرنشتاين).

اخيرا، كتطبيق على كثيرات حدود بيرنشتاين في التقريب، تمت دراسة منحنيات بيزيير والتي تستخدم بشكل واسع في الرسم الحاسوبي في تمثيل المنحنيات الملساء.

ABSTRACT

ON BERNSTEIN POLYNOMIALS AND APPROXIMATION

Shahrazad Naser AL-Dalaeen

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In this thesis some properties of Bernstein polynomials have been studied and justified. Such properties have been used for building the proof of weierstrass theorem in details. Extra results are obtained regarding Bernstein polynomials, which are used to introduce Jackson theorem. Similar result is obtained by replacing the trigonometric polynomials in Jackson theorem by (Bernstein polynomials).

Finally, as an application of Bernstein polynomials in approximation, Bézier curves are studied which are widely used in computer graphics to model smooth curves.